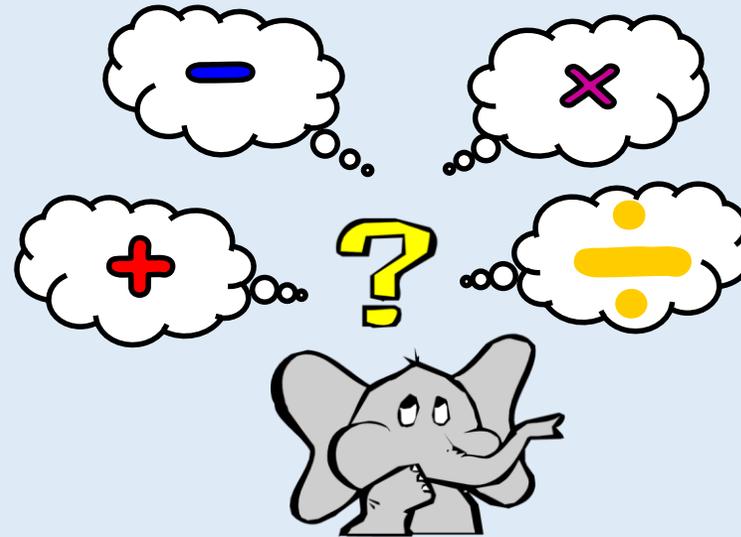


The Nar Valley Mathematics Calculation Policy



Commissioned by The PiXL Club Ltd.
June 2016

ADAPTED FOR USE AT THE NAR VALLEY FEDERATION OF SCHOOLS BY E. WILLGRESS – JANUARY 2020

This resource is strictly for the use of member schools for as long as they remain members of The PiXL Club. It may not be copied, sold nor transferred to a third party or used by the school after membership ceases. Until such time it may be freely used within the member school. All opinions and contributions are those of the authors. The contents of this resource are not connected with nor endorsed by any other company, organisation or institution.

About Nar Valley's PiXL Calculation Policy



- The following calculation policy has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school.

- Age stage expectations:

The calculation policy is organised according to age stage expectations as set out in the National Curriculum 2014 and the method(s) shown for each year group should be modelled to the vast majority of pupils. However, it is vital that pupils are taught according to the pathway that they are currently working at and are showing to have 'mastered' a pathway before moving on to the next one. Of course, pupils who are showing to be secure in a skill can be challenged to the next pathway as necessary.

- Choosing a calculation method:

Before pupils opt for a written method they should first consider these steps:

Can I do it in my head using a mental strategy?



Could I use some jottings to help me?



Should I use a formal written method to work it out?

Calculation Guidance Principles

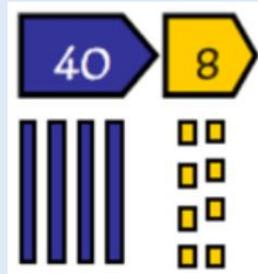
- *Develop children's fluency with basic number facts*
- *Develop children's fluency in mental calculation*
- *Develop children's understanding of the = symbol*
- *Teach inequality alongside teaching equality*
- *Use empty box problems*
- *Use intelligent practice*
- *Expose mathematical structure and work systematically*
- *Move between the concrete and the abstract*
- *Contextualise the mathematics*

Addition

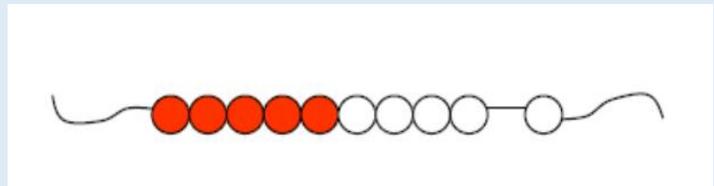
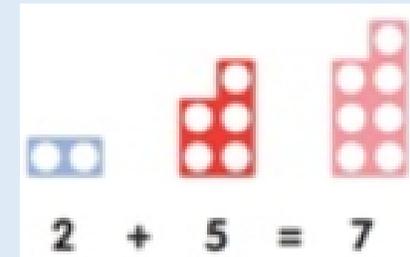


Concrete resources:

- 100 square
- Number lines
- Bead strings
- Straws
- Dienes
- Place value cards
- Place value dice
- Place value counters
- Numicon



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



sum addition total

make more

and plus

add increase

altogether

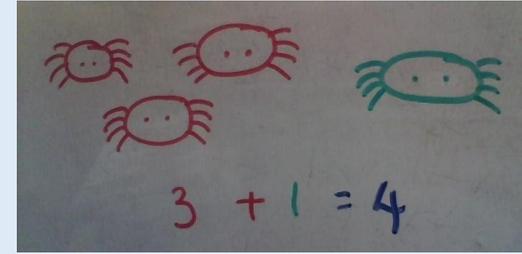
+

Addition: Reception



Early learning goals:

- ✓ Count reliably with numbers from 1 to 20, place them in order.
- ✓ Say which number is one more than a given number.
- ✓ Using quantities and objects, they add two single-digit numbers and count on to find the answer.



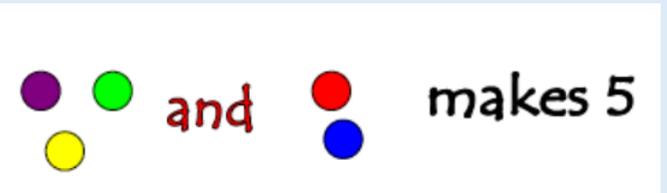
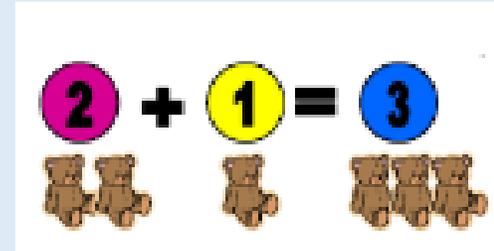
Recognise numbers up to 20 and understand the meaning of each number by recognising and knowing their clusters

1 one 	2 two 	3 three
4 four 	5 five 	6 six
7 seven 	8 eight 	9 nine

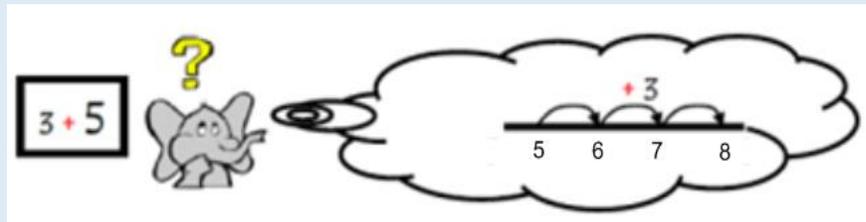
Count on in ones and say which number is one more than a given number using a number line or number track to 20.

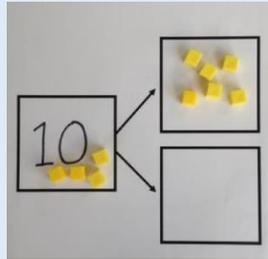


Begin to relate addition to combining two groups of objects using practical resources, role play, stories and songs.



Know that counting on is a strategy for addition. Use numbered number lines to 20.



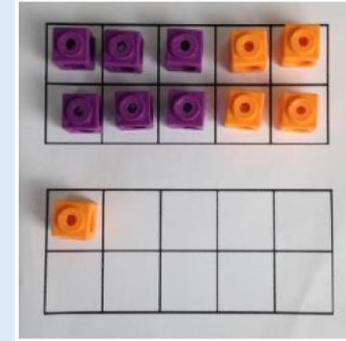


Addition: Year 1



Year 1 statutory requirements :

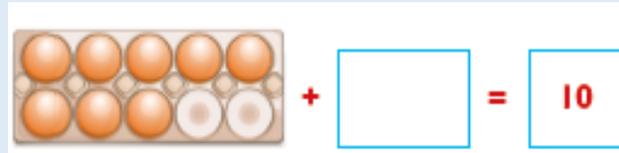
- ✓ Count to and across 100, forwards beginning with 0 or 1, or from any given number.
- ✓ Given a number, identify one more.
- ✓ Read, write and interpret mathematical statements involving addition (+), and equals (=) signs.
- ✓ Represent and use number bonds and related subtraction facts within 20
- ✓ Add one-digit and two-digit numbers to 20, including zero.
- ✓ Solve one-step problems that involve addition using concrete objects and pictorial representations, and missing number problems.



Identify and represent numbers using objects and pictorial representations (multiple representations)



Memorise and reason with number bonds to 10 and 20 in several forms.



Count on in ones to and across 100 and find one more than a given number.

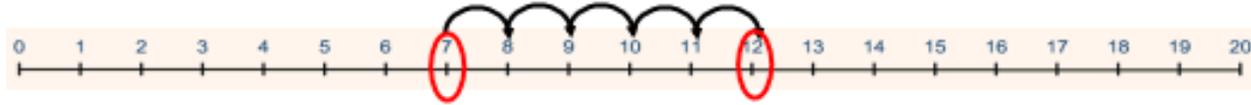
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

	$6 + 4 = 10$ $4 + 6 = 10$ $10 - 4 = 6$ $10 - 6 = 4$		<table border="1"> <tr><td colspan="2">10</td></tr> <tr><td>6</td><td>4</td></tr> </table>	10		6	4
10							
6	4						
Tens Frame		Part Whole Model					
		$6 + 4 = 10$ $4 + 6 = 10$ $10 - 4 = 6$ $10 - 6 = 4$					
		Bar Model					

Addition: Year 1

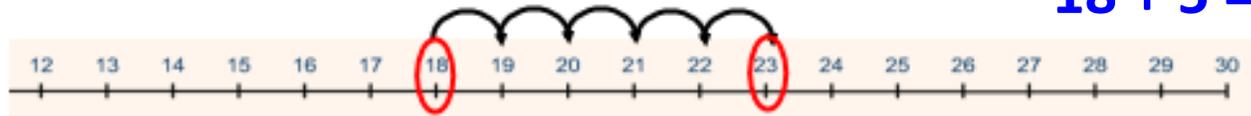


Use concrete resources and a number line to support the addition of numbers. Know and use strategy of finding the larger number, and counting on in ones from this number.



1 digit + 1 digit

$$7 + 5 = 12$$



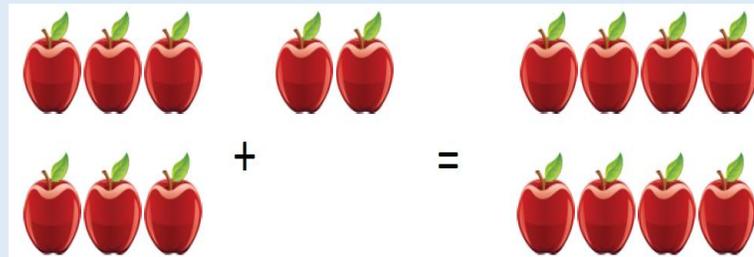
2 digit + 1 digit

$$18 + 5 = 23$$

Begin to use the + and = signs to write calculations in a number sentence.

Solve one-step problems using concrete objects and pictorial representations.

*Tom picks 6 apples and Raj picks 2 apples.
How many apples do they have altogether?*

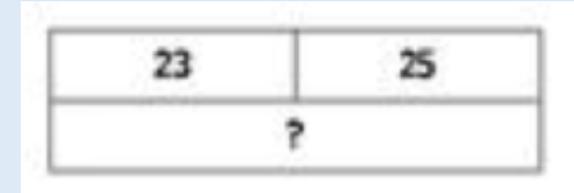


Addition: Year 2



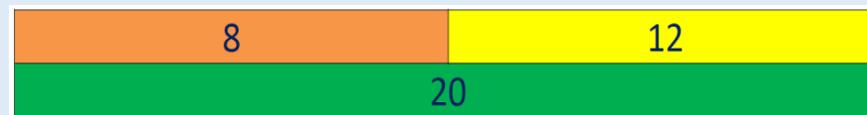
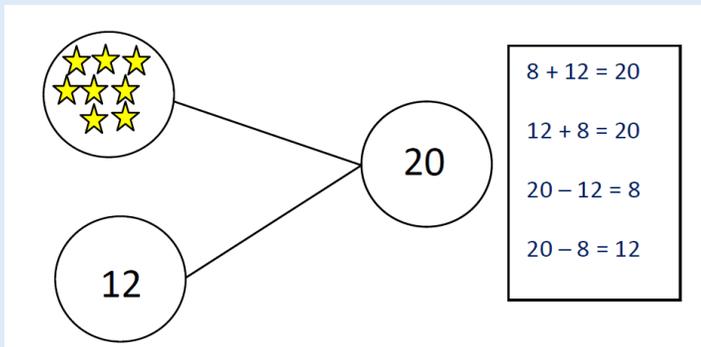
Year 2 statutory requirements :

- ✓ Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts to 100.
- ✓ Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.
- ✓ Add numbers using concrete objects, pictorial representations, and mentally, including:
 - a two-digit number and ones
 - a two-digit number and tens
 - two two-digit numbers
 - adding three one-digit numbers.
- ✓ Solve problems with addition including those involving numbers, quantities and measures.



Partition two 2-digit numbers using a variety of models and images.

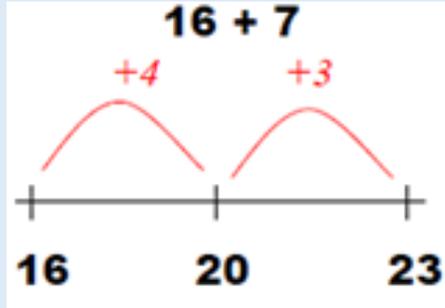
Memorise and reason with number facts to 20 in several forms.



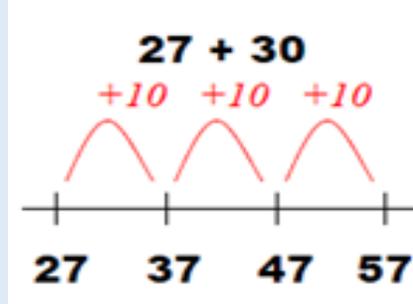
Addition: Year 2



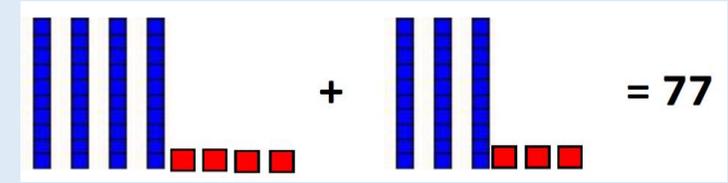
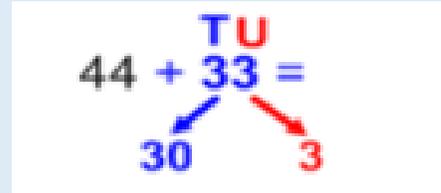
Add 2 digit number and ones



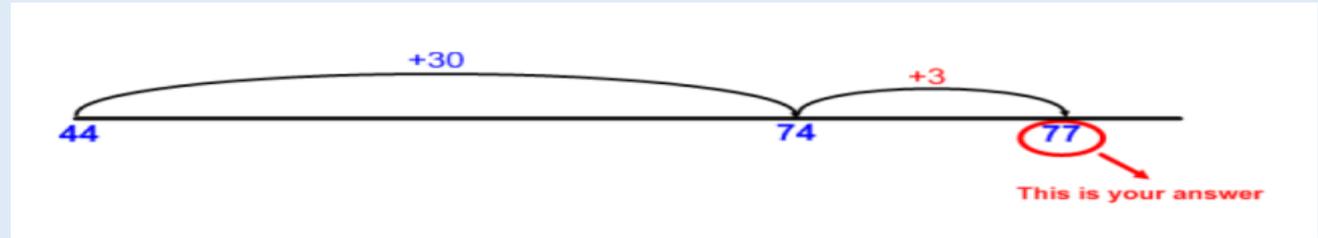
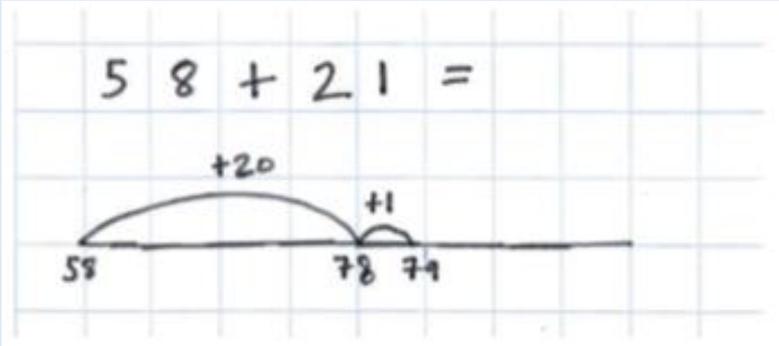
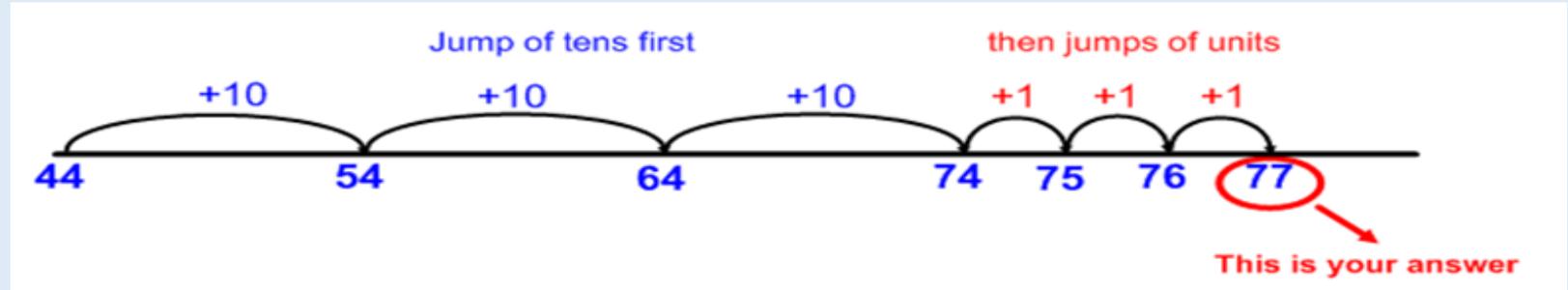
Add 2 digit number and tens



Use partitioning to add two 2-digit numbers using concrete resources and/or a numbered number line and then progressing to an empty number line.

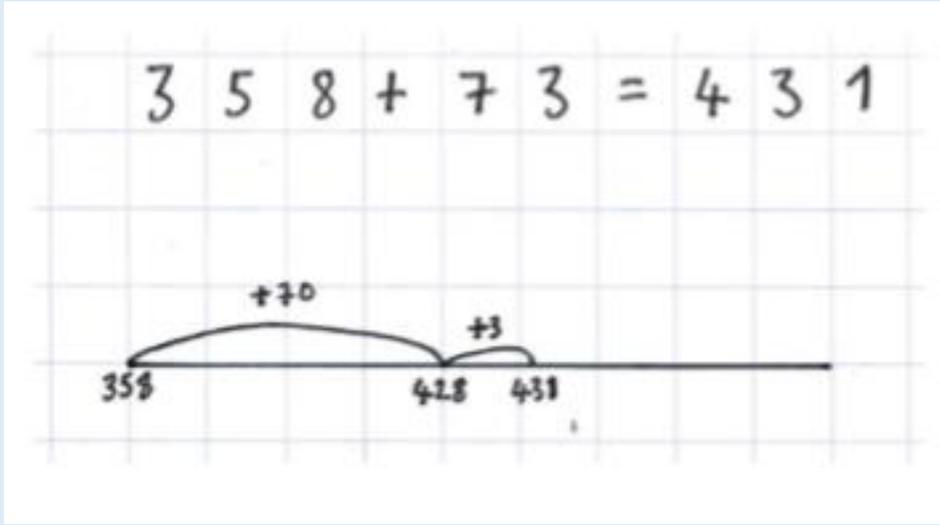


As children gain confidence with adding on tens and ones, they should be taught to combine the jumps on an empty number line. Add the tens first, then the ones. Taught with Dienes or place value counters alongside.





Continue to add using number lines.



Use expanded column method with place value resources to support the conceptual understanding of adding numbers up to three digits **with no carrying**.

$$42 + 31 = 73$$

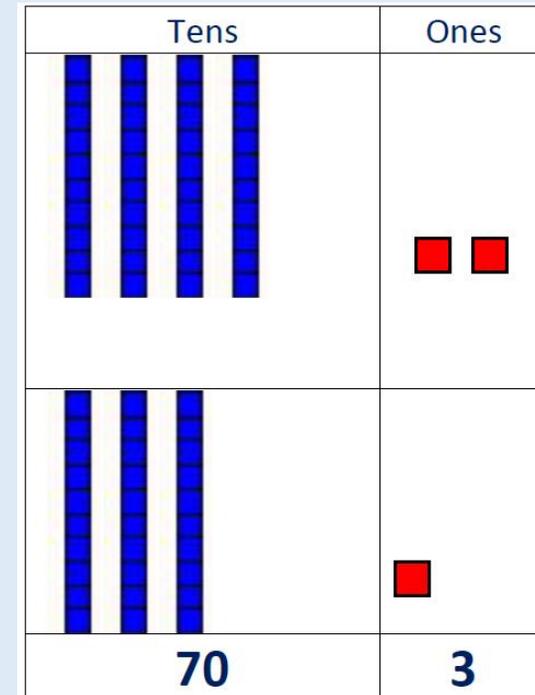
$$40 + 2$$

$$\underline{30 + 1}$$

$$\underline{70 + 3}$$



OR



Addition: Year 3



Year 3 statutory requirements :

- Find 10 or 100 more than a given number.
- Recognise the place value of each digit in a three-digit number (hundreds, tens, ones).
- Add numbers with up to three digits, using formal written methods of columnar addition.



Addition: Year 3



Progress to using the expanded column method with place value resources to support the conceptual understanding of adding numbers up to three digits **with carrying**.

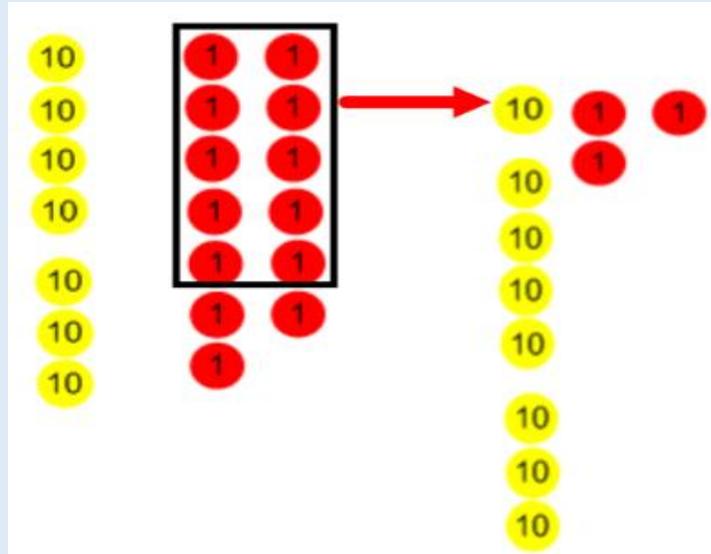
$$47 + 36 = 83$$

$$40 + 7$$

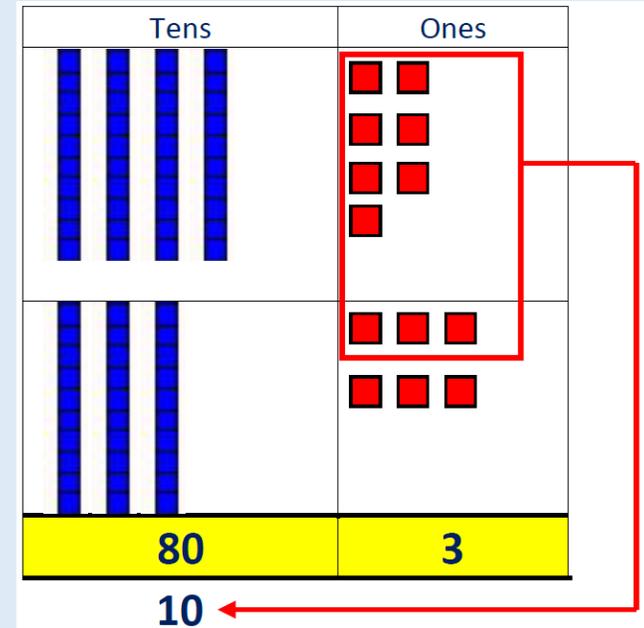
$$30 + 6$$

$$80 + 3$$

$$10$$



OR



Extend to using the expanded column method to add three digit numbers + three digit numbers **with carrying**.

$$367 + 185 = 552$$

$$300 + 60 + 7$$

$$100 + 80 + 5$$

$$500 + 50 + 2$$

$$100 \quad 10$$

Note: The carried ten or carried hundred is just as important as any other number, therefore, it should be written as clear and as large as any other number, and placed at the **bottom** of the column in which it is to be added.

Addition: Year 4



Year 4 statutory requirements :

- Find 1000 more than a given number.
- Add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate.
- Solve addition two-step problems in contexts, deciding which operations and methods to use and why,

Build on learning from Year 3 and model how expanded method links to compact column addition method.

$$\begin{array}{r}
 40 + 7 \\
 \underline{30 + 6} \\
 \underline{80 + 3} \\
 10
 \end{array}
 \rightarrow
 \begin{array}{r}
 47 \\
 \underline{+36} \\
 83 \\
 1
 \end{array}$$

$$\begin{array}{r}
 300 + 60 + 7 \\
 \underline{100 + 80 + 5} \\
 \underline{500 + 50 + 2} \\
 100 \quad 10
 \end{array}
 \rightarrow
 \begin{array}{r}
 367 \\
 \underline{+185} \\
 \underline{552} \\
 11
 \end{array}$$

Note: The carried ten or carried hundred is just as important as any other number, therefore, it should be written as clear and as large as any other number, and placed at the **bottom** of the column in which it is to be added.

By the end of year 4, pupils should be adding numbers up to 4 digits using the compact column addition method.

$$\begin{array}{r}
 5271 \\
 + 2357 \\
 \hline
 7628 \\
 1
 \end{array}$$

Addition: Year 5 & 6



Year 5 statutory requirements :

- Add whole numbers with more than 4 digits using formal written methods of columnar addition.
- Add numbers mentally, with increasingly large numbers.
- Solve addition multi-step problems in contexts, deciding which operations and methods to use and why.
- Solve problems involving numbers up to three decimal places

Year 6 statutory requirements :

- Pupils are expected to solve more complex addition and subtraction problems

In year 5 and 6 pupils should be adding numbers using compact column addition method. **Note:** The carried ten, hundred, thousand is just as important as any other number, therefore, it should be written as clear and as large as any other number, and placed at the **bottom** of the column in which it is to be added.

$$\begin{array}{r}
 46892 \\
 + 32758 \\
 \hline
 79650 \\
 \hline
 111
 \end{array}$$

When adding decimals, it is essential that the decimal point does not move and is kept in line.

Where necessary, a zero should be added as a **place holder**.

$$12.5 + 23.7$$

$$\begin{array}{r}
 12.5 \\
 + 23.7 \\
 \hline
 36.2 \\
 \hline
 1
 \end{array}$$

$$34.5 + 27.43$$

$$\begin{array}{r}
 34.50 \\
 + 27.43 \\
 \hline
 61.93 \\
 \hline
 1
 \end{array}$$

Subtraction

Concrete resources:

- 100 square
- Number lines
- Bead strings
- Straws
- Dienes
- Counting stick
- Place value dice
- Place value cards
- Place value counters



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



subtract

count on count back

fewer — less

take away minus

 difference





Subtraction: Reception



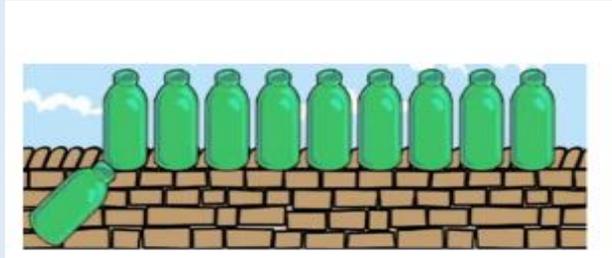
Early learning goals:

- ✓ Say which number is one less than a given number.
- ✓ Using quantities and objects, they subtract two single-digit numbers and count back to find the answer.

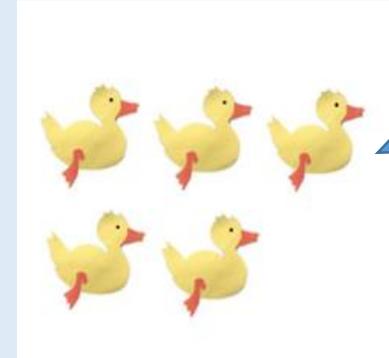
Say which number is one less than a given number using a number line or number track to 20.



Begin to count backwards in familiar contexts such as number rhymes or stories.



10 Green Bottles sitting on the wall ...



5 little ducks went swimming one day...

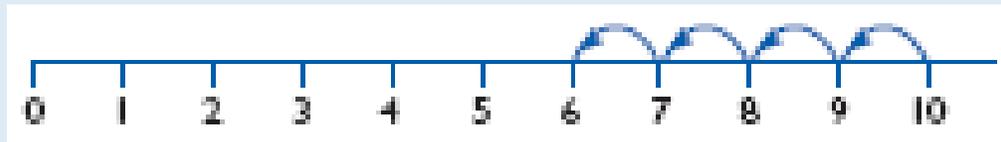
Begin to relate subtraction to 'taking away' using concrete objects and role play.



Three teddies **take away** two teddies leaves one teddy



If I **take away** four shells there are six left



Count backwards along a number line to 'take away'

Subtraction: Year 1

PIXL
PRIMARY

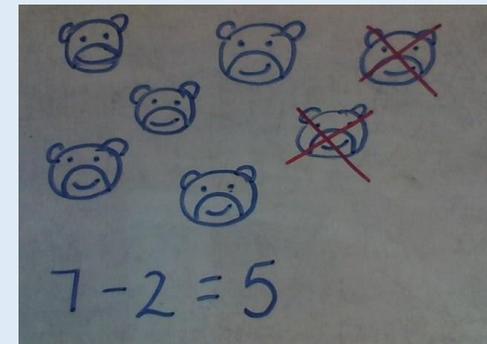


Maths



Year 1 statutory requirements:

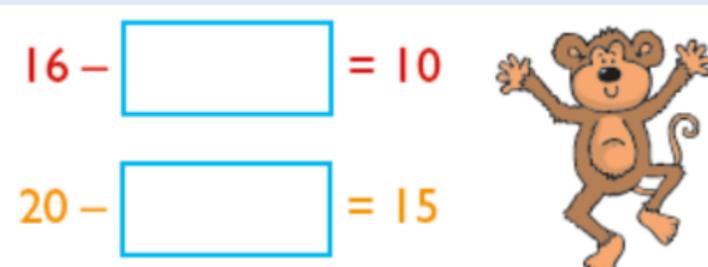
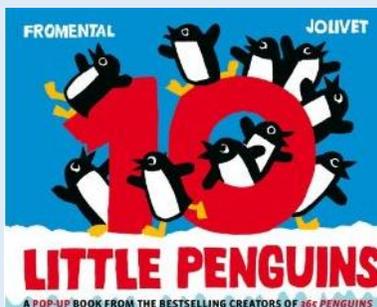
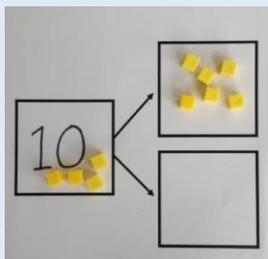
- ✓ Say which number is one less than a given number.
- ✓ Represent and use number bonds and related subtraction facts within 20.
- ✓ Read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs.
- ✓ Subtract one-digit and two-digit numbers to 20, including zero.
- ✓ Solve one-step problems that involve subtraction using concrete objects and pictorial representations, and missing number problems.



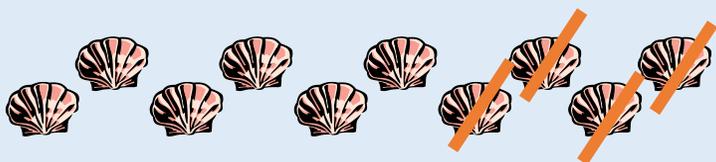
Understand subtraction as taking away. Use practical resources, pictorial representations, role play, stories and rhymes.

Use number bonds and related subtraction facts within 20.

Count back in ones and find one less than a given number.



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

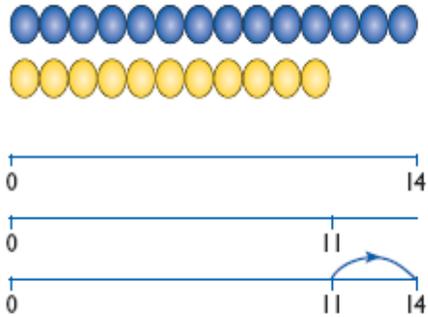


<p>6 + 4 = 10 4 + 6 = 10 10 - 4 = 6 10 - 6 = 4</p> <p>Tens Frame</p>	<p>6 + 4 = 10 4 + 6 = 10 10 - 4 = 6 10 - 6 = 4</p> <p>Part Whole Model</p>	<table border="1"> <tr><td colspan="2">10</td></tr> <tr><td>6</td><td>4</td></tr> </table> <p>6 + 4 = 10 4 + 6 = 10 10 - 4 = 6 10 - 6 = 4</p> <p>Bar Model</p>	10		6	4
10						
6	4					

Subtraction: Year 1

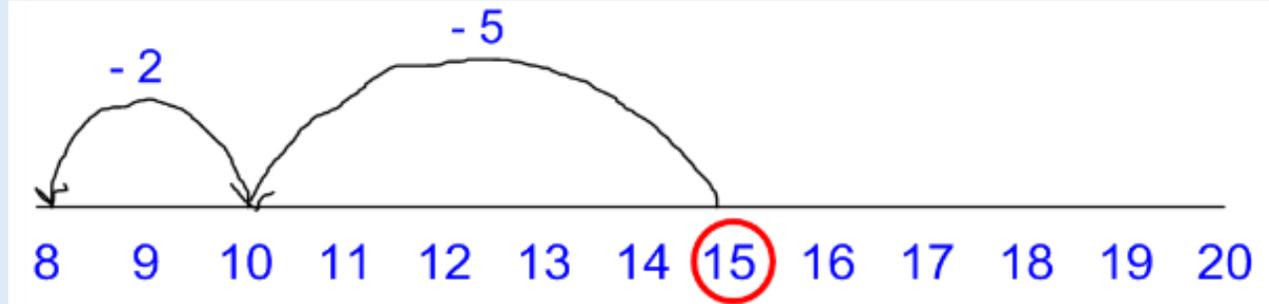


Counting on should only be used when the language used is 'find the difference', 'difference between' and 'distance between'.



The difference between 11 and 14 is 3.
 $14 - 11 = 3$
 $11 + \square = 14$

Use number line to support the subtraction of numbers. Know and use strategy of **counting back** to subtract one-digit and two-digit numbers to 20.



$$15 - 7 = 8$$

Solve one-step problems using concrete objects and pictorial representations.

Dan has 12 football stickers.
 He gives 4 to Ben.
 How many stickers does he have left?



$$12 - 4 = 8$$

Begin to use the - and = signs to write calculations in a number sentence.

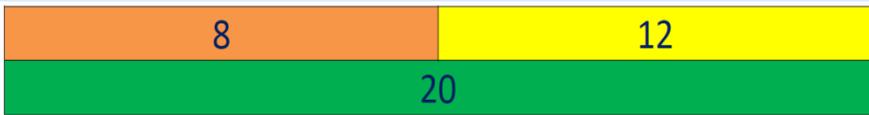
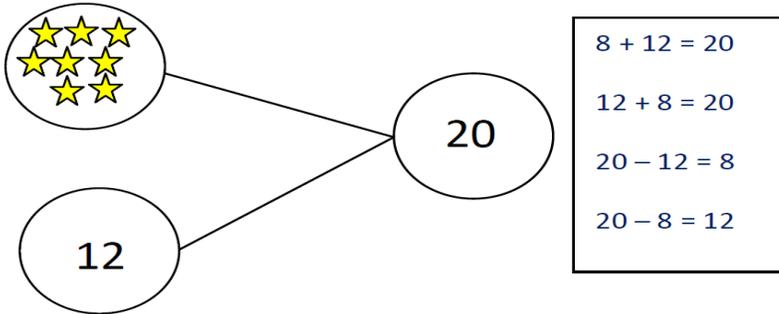
Subtraction: Year 2

Year 2 statutory requirements:

- Recall and use subtraction facts to 20 fluently, and derive and use related facts to 100.
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.
- Subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers; adding three one-digit numbers.

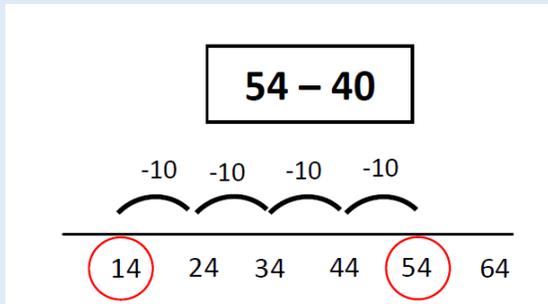
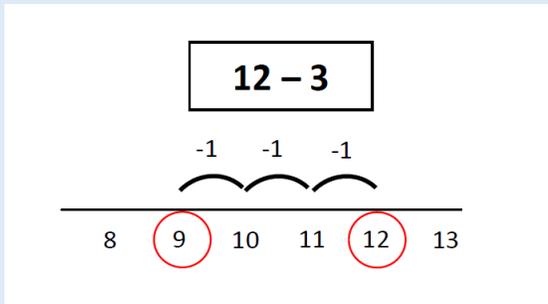
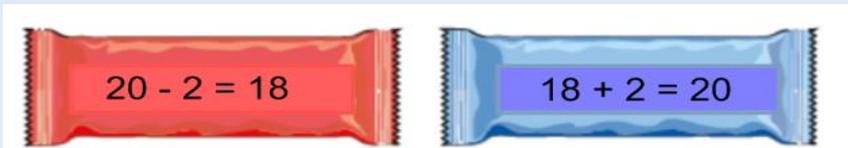
Memorise and reason with number facts to 20 in several forms.

Partition two 2-digit numbers using a variety of models and images.



Subtract 2 digit and ones

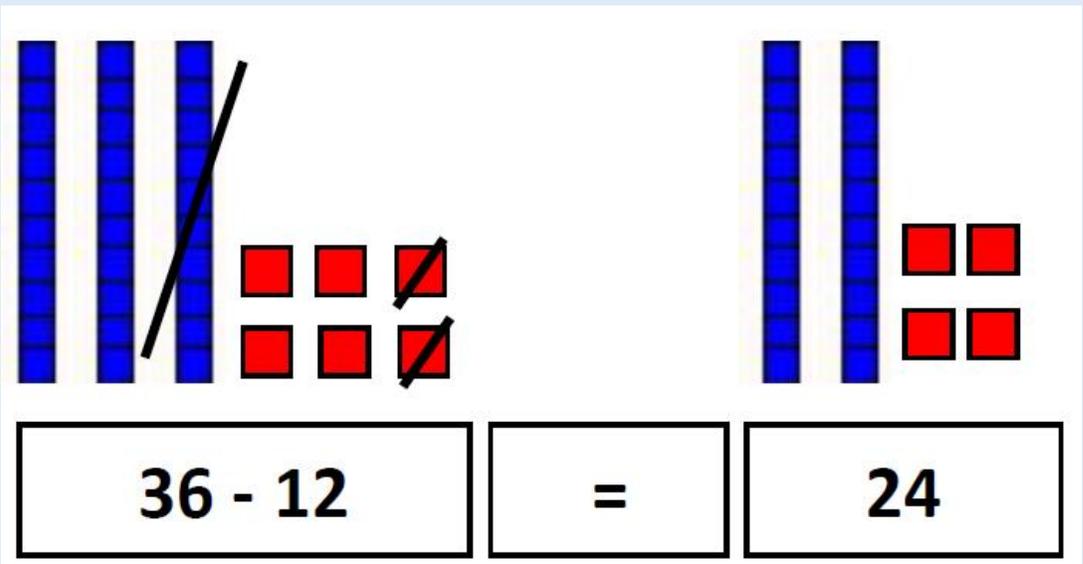
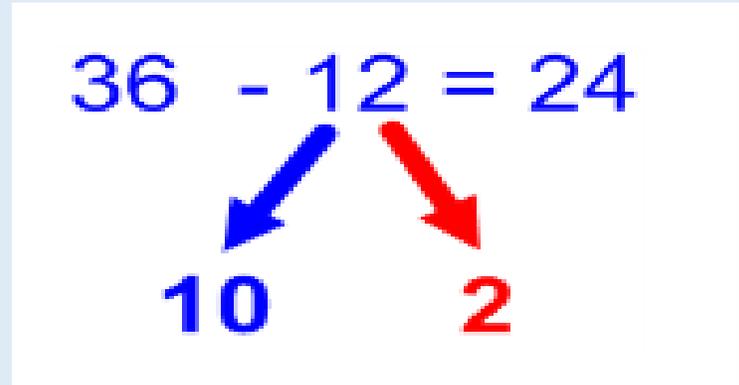
Subtract 2 digit and tens



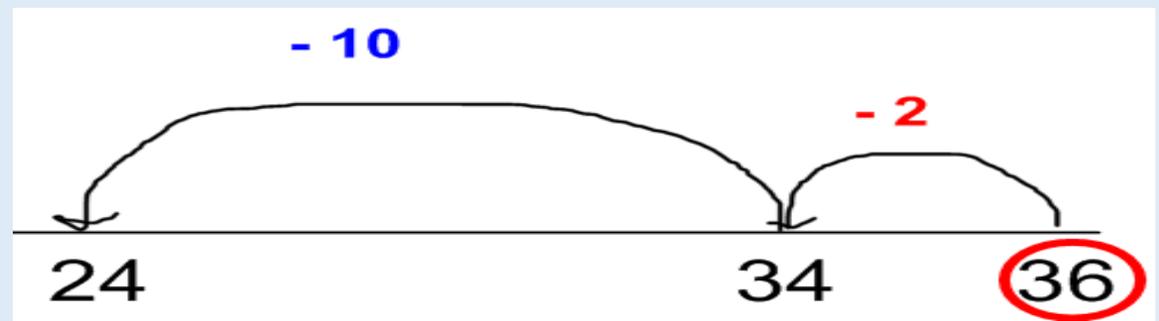
Subtraction: Year 2



Use partitioning to subtract two 2-digit numbers using concrete resources and/or a numbered number line and then progressing to an empty number line.



OR

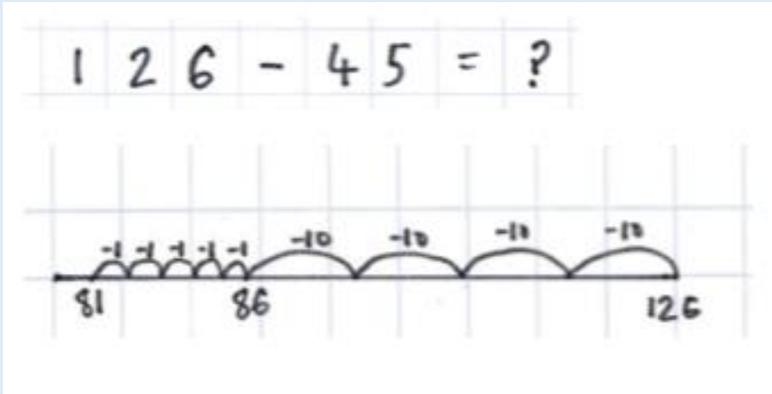


Subtraction: Year 3

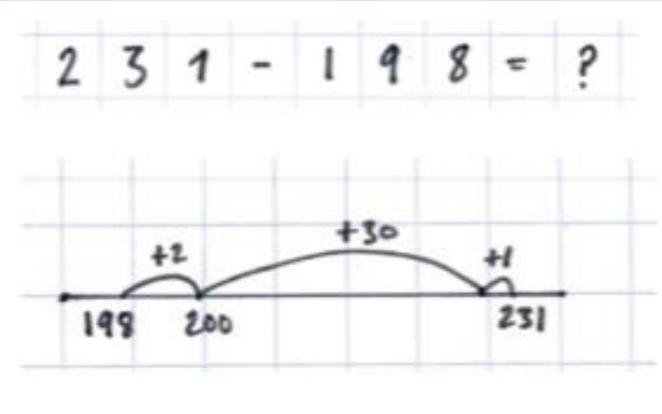


Year 3 statutory requirement:

- Find 10 or 100 less than a given number.
- Recognise the place value of each digit in a three-digit number (hundreds, tens, ones).
- Subtract numbers with up to three digits, using formal written methods of column subtraction.
- Subtract numbers mentally, including: a 3-digit number & ones; a 3-digit number & tens; a 3-digit number & hundreds.



Counting back on a number line.
Counting back tens, then ones.

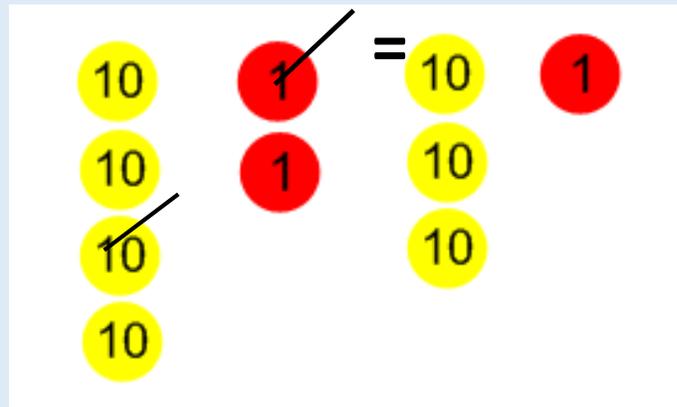


Finding a difference between two numbers by counting on.

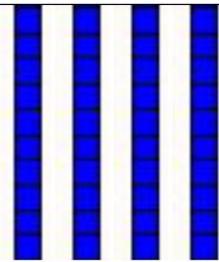
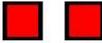
Use expanded column method with place value resources to support the conceptual understanding of subtracting numbers with up to three digits **with no exchanging**.

$$42 - 11 = 31$$

$$\begin{array}{r}
 40 + 2 \\
 - 10 + 1 \\
 \hline
 30 + 1
 \end{array}$$



OR

Tens	Ones
	
- 10	- 1
30	1



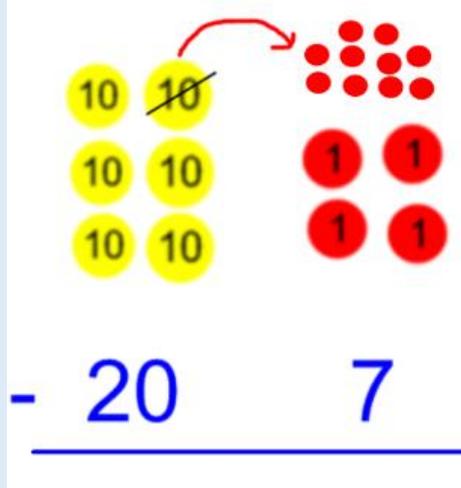
Subtraction: Year 3



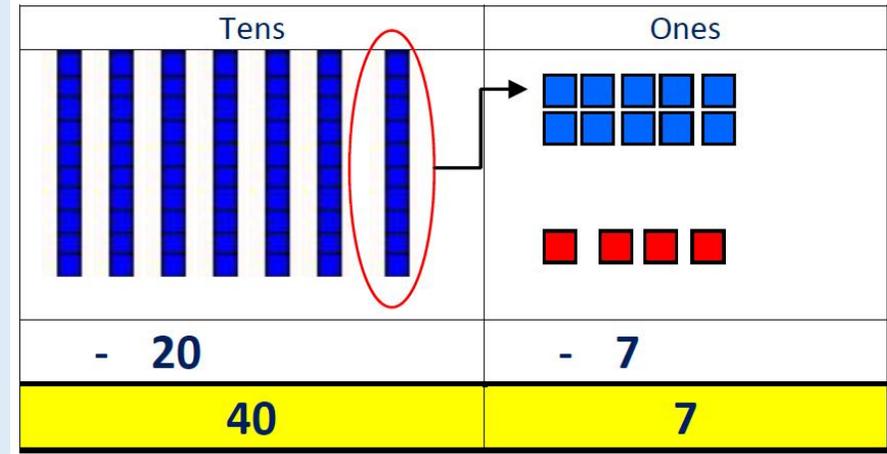
Progress to using the expanded column method with place value resources to support the conceptual understanding of subtracting numbers with up to three digits **with exchanging tens and/or hundreds**.

$$74 - 27 = 47$$

$$\begin{array}{r} 60 + 14 \\ 70 + 4 \\ - 20 + 7 \\ \hline 40 + 7 \end{array}$$



OR



In this example to subtract 7 ones from 4 ones we need to **exchange** a ten for ten ones. We now can subtract 7 ones from 14 ones.

Extend to using the expanded column method to subtract three digit numbers from three digit numbers.

$$537 - 254 = 283$$

$$\begin{array}{r} 400 + 130 \\ 500 + 30 + 7 \\ - 200 + 50 + 4 \\ \hline 200 + 80 + 3 \end{array}$$

Note: The exchanged ten or hundred is just as important as any other number, therefore, it should be written as clear and as large as any other number, and placed at the **top** of the column which has been adjusted.

Subtraction: Year 4



Build on learning from Year 3 and model how expanded method links to compact column subtraction method.

Year 4 statutory requirements:

- Find 1000 less than a given number.
- Subtract numbers with up to four digits, using formal written methods of columnar subtraction where appropriate.
- Solve subtraction two-step problems in contexts, deciding which operations and methods to use and why.

$$\begin{array}{r}
 60 + 14 \\
 \cancel{70} + \cancel{4} \\
 - \quad \quad \\
 \hline
 20 + 7 \\
 \hline
 40 + 7
 \end{array}$$



$$\begin{array}{r}
 6 \\
 \cancel{7}^1 4 \\
 - \quad 27 \\
 \hline
 47
 \end{array}$$

$$\begin{array}{r}
 400 + 130 \\
 \cancel{500} + \cancel{30} + 7 \\
 - \quad \quad \quad \\
 \hline
 200 + 50 + 4 \\
 \hline
 200 + 80 + 3
 \end{array}$$



$$\begin{array}{r}
 4 \\
 \cancel{5}^1 \cancel{3}^1 7 \\
 - \quad \quad \quad \\
 \hline
 254 \\
 \hline
 283
 \end{array}$$

By the end of year 4, pupils should be subtracting numbers up to 4 digits using compact column subtraction method.

$$\begin{array}{r}
 3 \\
 78\cancel{4}^1 2 \\
 - \quad \quad \quad \\
 \hline
 1829 \\
 \hline
 6013
 \end{array}$$

Note: The exchanged ten or hundred is just as important as any other number, therefore, it should be written as clear and as large as any other number, and placed at the **top** of the column which has been adjusted.

Subtraction: Year 5 & 6



Year 5 statutory requirements :

- Subtract whole numbers with more than 4 digits using formal written methods of columnar subtraction.
- Subtract numbers mentally, with increasingly large numbers.
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- Solve problems involving numbers up to three decimal places.

Year 6 statutory requirements: pupils are expected to solve more complex addition and subtraction problems

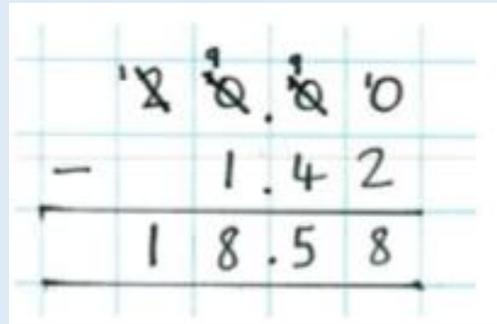
In year 5 and 6 pupils should be subtracting numbers using compact column subtraction method.

When subtracting decimals, it is essential that the decimal point does not move and kept in line.

Where necessary, a zero should be added as a **place holder**.

$$\begin{array}{r}
 41 \\
 \cancel{5} \cdot 37 \\
 - 2.54 \\
 \hline
 2.83
 \end{array}$$

$$\begin{array}{r}
 8 7 \\
 \cancel{9} ^1 6 7 \cancel{8} ^1 3 \\
 - \\
 \hline
 5 8 7 3 5 \\
 3 8 0 4 8
 \end{array}$$



	1	0	5	.	4	1	9	kg
-			3	.	0	8	0	kg
			6	.	3	3	9	kg



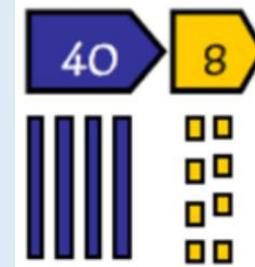
Concrete resources

- Place value counters
- Dienes
- Place value charts
- Arrays
- Multiplication squares
- 100 square
- Number lines
- Blank number lines
- Counting stick

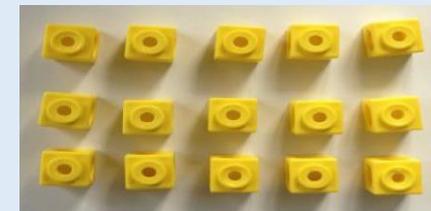


Multiplication

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100



multiplication **product**
 once, twice, three times
double **groups of**
 repeated addition **lots of**
 array, row, column **multiply**
times **multiple**

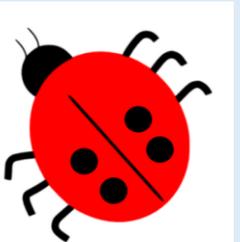
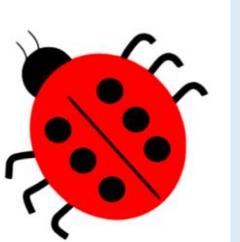
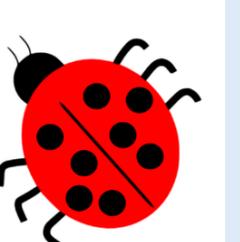


Multiplication: Reception



Early learning goal statutory requirement:
✓They solve problems, including doubling, halving and sharing.

Use pictorial representations and concrete resources to double numbers to 10.

			
$1 + 1 = 2$	$2 + 2 = 4$	$3 + 3 = 6$	$4 + 4 = 8$

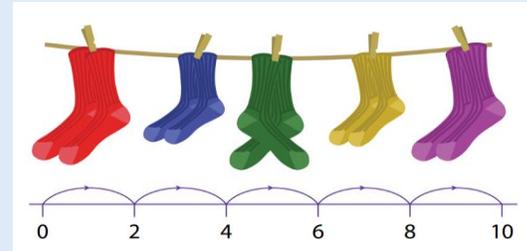
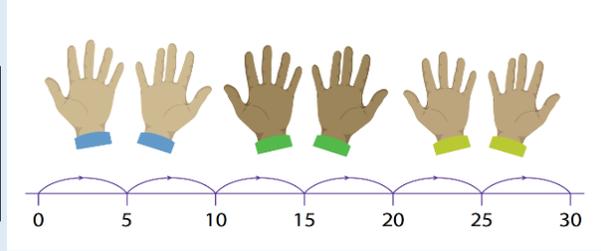
Use concrete sources, role play, stories and songs to begin counting in twos, fives and tens.

Multiplication: Year 1



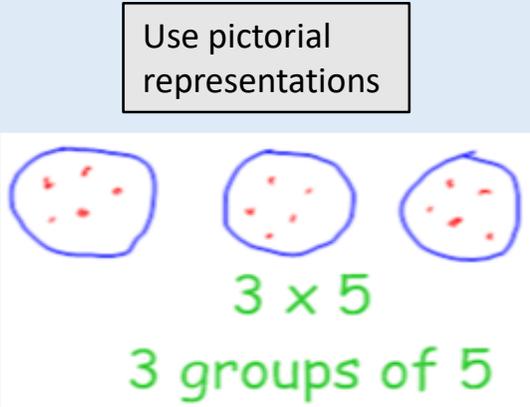
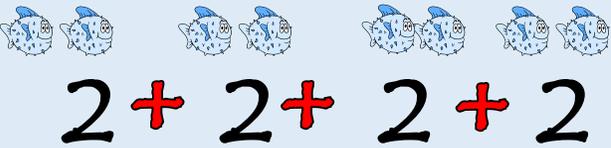
Year 1 statutory requirement:
 ✓ Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Count in twos, fives and tens using practical resources, role play, stories and songs.

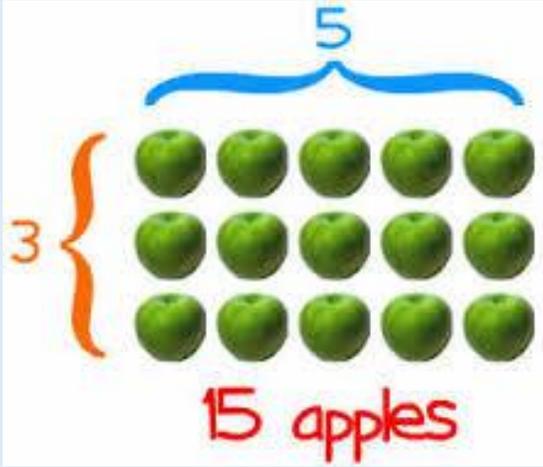


Use arrays

Understand multiplication as repeated addition – use concrete objects to support understanding.



Use pictorial representations



Multiplication: Year 2



Year 2 statutory requirement:

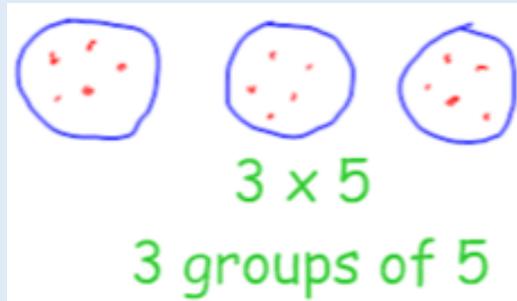
- ✓ Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- ✓ Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals ($=$) signs.
- ✓ Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- ✓ Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Further develop understanding of multiplication as repeated addition.

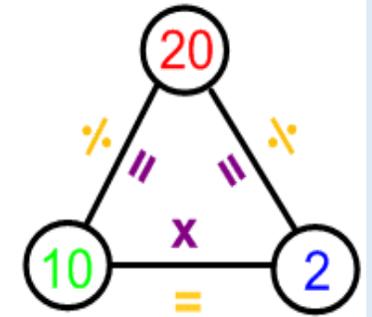
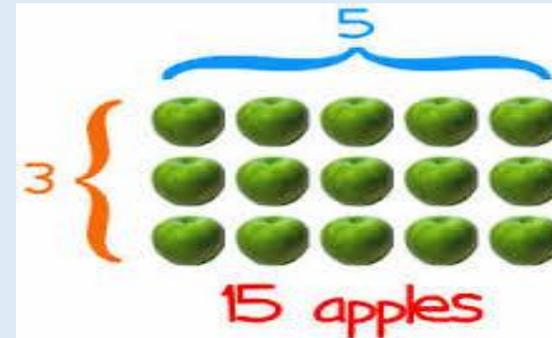


$5 + 5 + 5$
or
 5×3

Use pictorial representations

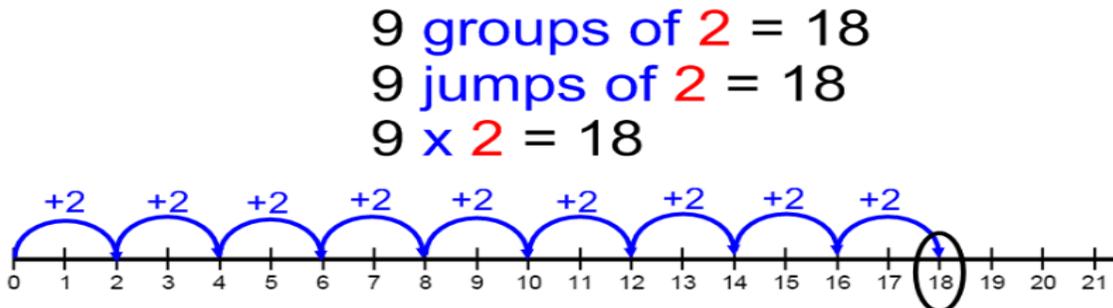


Use arrays



Recall multiplication and division facts for 2, 5, 10

Model and bridge link from repeated addition to solving multiplication problems using a number line.

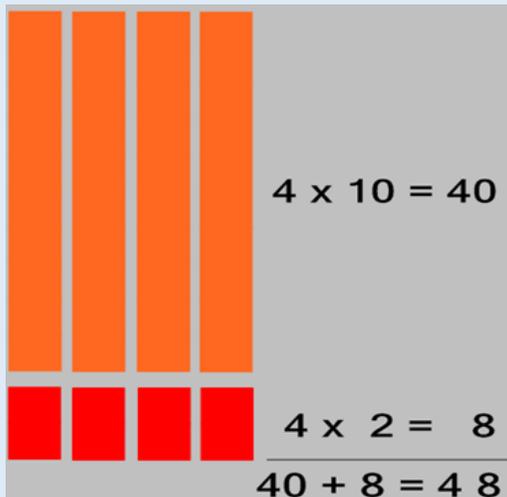




Develop recall of multiplication facts (alongside inverse of the corresponding division facts).

x	3	4	8	x	4	?	?
5				?	8	6	10
6				6	24	18	30
4				?	32	24	40

$12 \times 4 = 48$



Multiplication: Year 3



Year 3 statutory requirements:

- ✓ Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.
- ✓ Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- ✓ Solve problems, including missing number problems, involving multiplication including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Use concrete resources and pictorial representations to develop conceptual understanding of the grid method of multiplication.

x	10	2
4		

OR

x	10	2
4	40	8



The yellow ribbon is 4 times as long as the red ribbon. What is its length?

Multiplication: Year 4

Year 4 statutory requirement:

- ✓ Recall multiplication and division facts for multiplication tables up to 12×12
- ✓ Use place value, known and derived facts to multiply and divide mentally, including: multiply two-digit and three-digit numbers by a one-digit number using formal written layout.
- ✓ Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Build on learning from Year 3, continue to use the grid method to multiply increasingly larger numbers.

$$18 \times 3 = 54$$

X	10	8
3	30	24

$$135 \times 5 = 675$$

X	100	30	5
5	500	150	25

$$24 \times 16 = 384$$

X	20	4
10	200	40
6	120	24

Develop recall of multiplication facts (alongside the inverse of the corresponding division facts).



2 eggs
150g flour
180g sugar

Use knowledge of times tables to solve scaling problems.

Susie wants to bake 12 cupcakes people.
The ingredients given are for four cupcakes.
How much flour will she need?



<u>Cupcakes</u>	<u>Flour</u>
4	150g
12	900g



Multiplication: Year 5

Year 5 statutory requirements:

- ✓ Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.
- ✓ Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

To multiply by 10, 100, 1000 children should use place value charts to show that the digit moves a column (s) to the left. The value of the digit is increasing by 10, 100 or 1000 times.

Progressing from grid method to short multiplication The short multiplication method is introduced alongside the grid method and the expanded form to aid understanding.

24 x 6 =

20	4
6	24

→

20	4
6	24
120	24
144	

→

24	
x 6	
144	

124 x 5 =

124	
x 5	
20	(4 x 5)
100	(20 x 5)
500	(100 x 5)
620	

→

124	
x 5	
620	

Th	H	T	U	1/10	1/100	1/1000
		3	6	.	4	2

x 10

Th	H	T	U	1/10	1/100	1/1000
		3	6	4	.	2

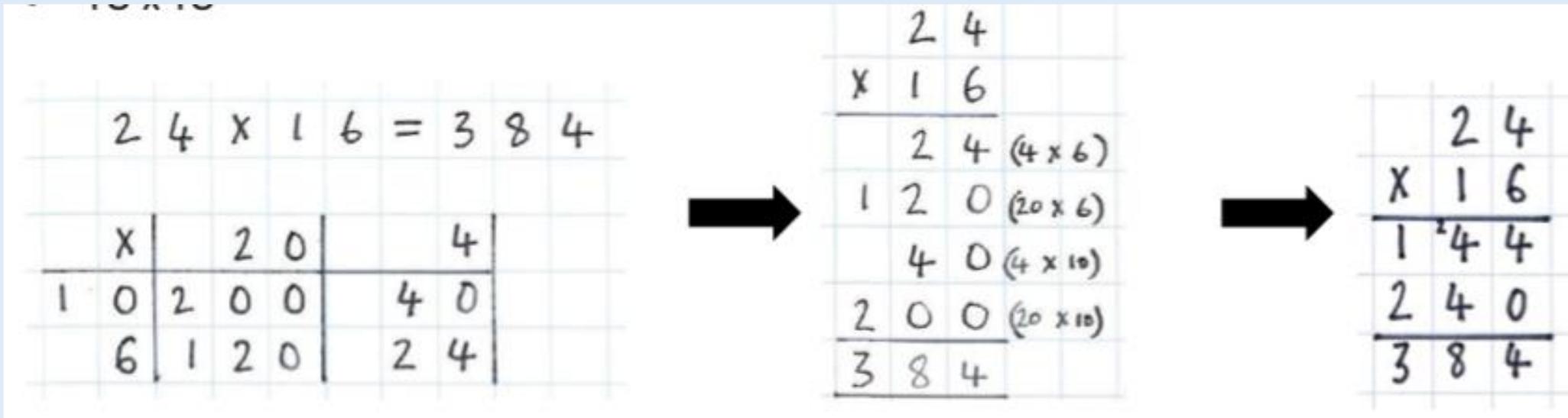
Multiplication: Year 5



Year 5 statutory requirements:

- ✓ Multiply numbers up to 4 digits by a 1- or 2-digit number using a formal written method, including long multiplication for two-digit numbers.
- ✓ Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

Progressing from grid method to long multiplication. When long multiplication is first introduced, both equations should be presented so that the answers to the individual multiplication steps are on the same line. Children should be encouraged to discuss what is similar and what is different.



The image shows three stages of multiplication on a grid background:

- Grid Method:** The equation $24 \times 16 = 384$ is written at the top. Below it, a grid is used to calculate the product. The grid is divided into three columns by vertical lines. The first column contains '10200' and '6120'. The second column contains '40' and '24'. The final result '384' is written to the right of the grid.
- Long Multiplication with Partial Products:** The equation $24 \times 16 = 384$ is written at the top. Below it, the multiplication is shown in a standard long multiplication format. The partial products are written on separate lines: 24 (from 4×6), 120 (from 20×6), 40 (from 4×10), and 200 (from 20×10). The final sum 384 is underlined.
- Formal Long Multiplication:** The equation $24 \times 16 = 384$ is written at the top. Below it, the multiplication is shown in a standard long multiplication format. The partial products are written on separate lines: 24 (from 4×6), 144 (from 20×6), 240 (from 4×10), and 384 (from 20×10). The final sum 384 is underlined.

Multiplication: Year 6



Year 6 statutory requirements:

- ✓ Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
- ✓ Multiply one-digit numbers with up to two decimal places by whole numbers.

Continue to use compact short multiplication to multiply by single digits

$$\begin{array}{r} 342 \\ \times 7 \\ \hline 2394 \\ \hline 21 \end{array}$$

$$\begin{array}{r} 2741 \\ \times 6 \\ \hline 16446 \\ \hline 42 \end{array}$$

Multiplying a two digit number by a three or four digit number should be introduced through the grid method before moving to long multiplication to aid understanding. When long multiplication is first introduced, both equations should be presented so that the answers to the individual multiplication steps are on the same line. Children should be encouraged to discuss what is similar and what is different.

Handwritten examples of multiplication methods on grid paper:

Equation: $262 \times 19 = 4,978$

Grid method (left):

		200	60	2	
10	2,000	600	20	2620	
9	1,800	540	18	2358	
				4,978	

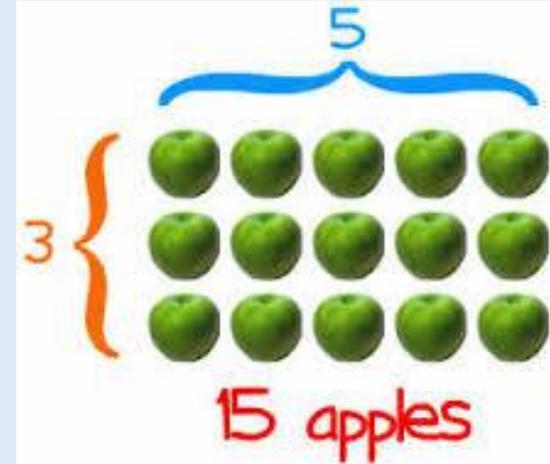
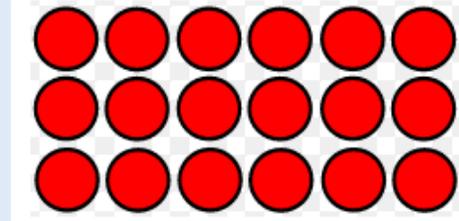
Grid method (right):

	262	
x	19	
	2358	
	2620	
	4,978	

Division

Concrete resources:

- Arrays
- Multiplication squares
- 100 square
- Number lines
- Blank number lines
- Counting stick
- Place value apparatus



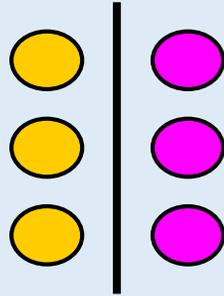
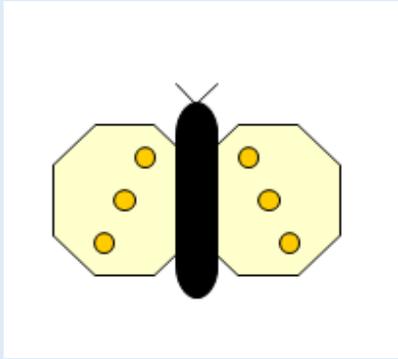
divided by group
 into lots of $\frac{\bullet}{-}{\bullet}$ into groups of
 divisible remainder halve
 half factor

Division: Reception



Early learning goal statutory requirement:
✓ They solve problems, including halving and sharing.

Use pictorial representations and concrete resources to halve numbers to 10.



Begin to share quantities using practical resources, role play, stories and songs.



Role play example:
It is the end of the party and the final two teddies are waiting for their party bags. Provide empty party bags and a small collection of items such as gifts, balloons and slices of cake. Ask the children to share the objects between the two bags.

Division: Year 1



Year 1 statutory requirement:

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Understand division as **sharing** using concrete resources.



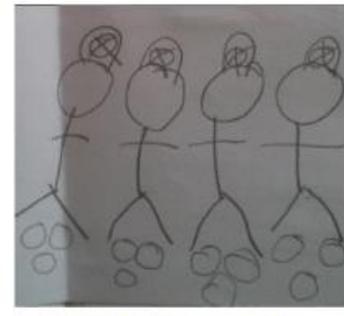
Pictorial representation of sharing **12 gold coins** between 2, 3 and 4 pirates!



$$12 \div 2$$



$$12 \div 3$$



$$12 \div 4$$

Begin to understand division as **grouping** using concrete resources.

12 into groups of 2

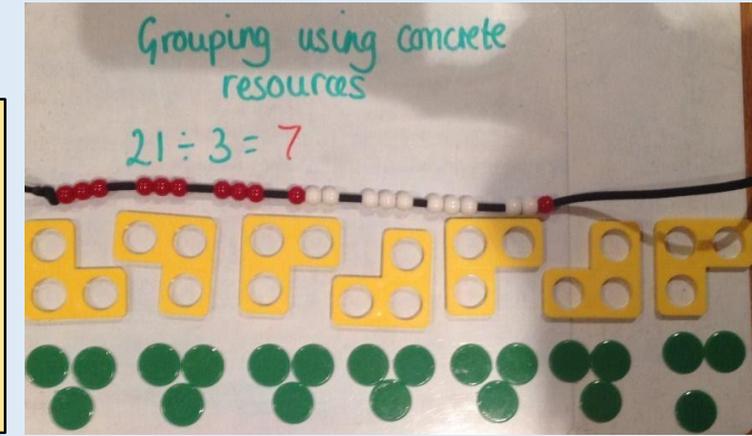
$$12 \div 2 = 6$$



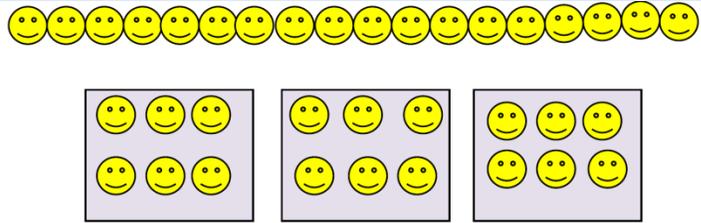
Division: Year 2

Year 2 statutory requirement:

- ✓ Recall and use division facts for 2, 5 and 10 multiplication tables.
- ✓ Calculate mathematical statements for multiplication and division within the multiplication tables and write then using the multiplication (x), division (÷) and equals (=) signs.
- ✓ Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.
- ✓ *Find 1/3; 1/4; 2/4; 3/4 of a length, shape, set of objects or quantity*

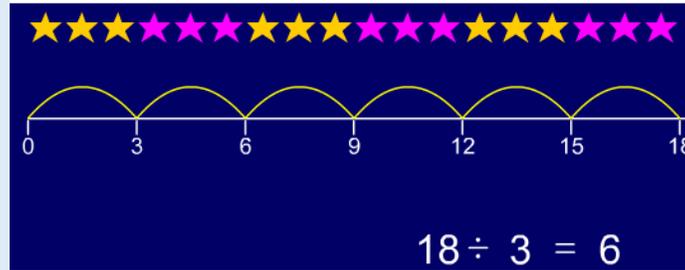


Further develop understanding of difference between **sharing** and **grouping** using concrete resources.



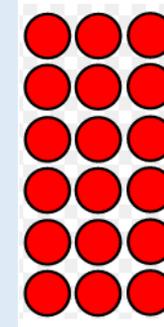
18 smiley faces shared between 3 classes.

Model division as grouping on a number line (ITP 'Grouping')



Children use numbered number lines to divide using grouping.

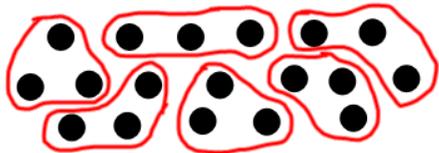
Reinforce division through the use of arrays.



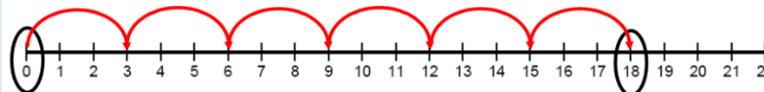
$$18 \div 3 = 6$$

$$18 \div 6 = 3$$

18 into groups of 3
 $18 \div 3 = 6$



18 into groups of 3 = 6 groups
18 into jumps of 3 = 6 jumps
 $18 \div 3 = 6$



Remember to develop connections between fractions and division and rephrase this calculation as 1/3 of 18 is the same as $18 \div 3 = 6$.



Division: Year 3 & 4



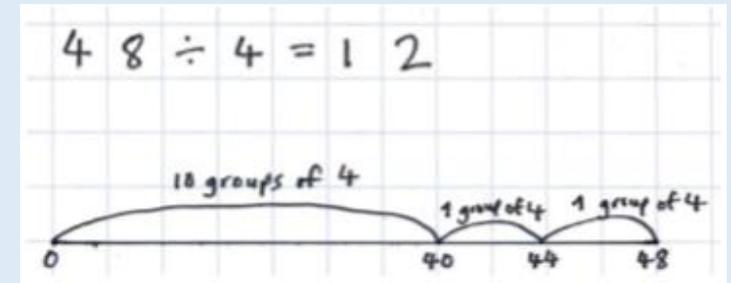
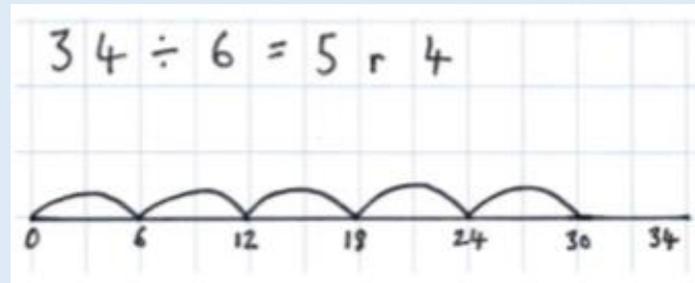
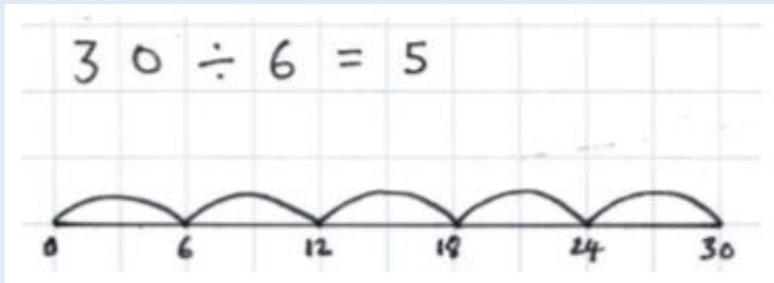
Year 3 statutory requirement:

- ✓ Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- ✓ Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- ✓ Solve problems, including missing number problems, involving division including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

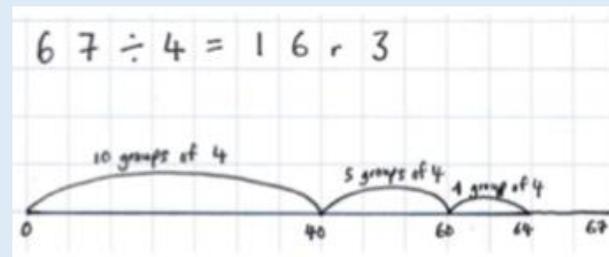
1. Grouping: A number line counting up from zero. This representation should be supported by grouping of concrete materials and other pictorial representations.

2. Grouping with remainders: Again using a number line counting up from zero. This should also be supported by grouping of concrete materials and other pictorial representations.

3. Efficient Grouping: Rather than counting individually, children now use groups for efficiency. The number of groups should be recorded above the jump.



4. Efficient Grouping with remainders: The efficient grouping method now incorporates remainders



See video link in 'notes' to consider how to develop conceptual understanding of division using dienes.

Division: Year 3 & 4*



Year 4 statutory requirement: **Note** - there isn't a statutory objective for division. However, Y4 statutory multiplication objectives are to (1) recall multiplication and division facts for multiplication tables up to 12×12 and (2) multiply two-digit and three-digit numbers by a one-digit number using formal written layout so we will build on the connections between multiplication and division.

In Year 4, continue to develop the use of informal number line methods taught in Year 3. Start to teach compact short division when children are secure with dividing on a number line. Teach compact short division using the following progression.

$$\begin{array}{r} 32 \\ 3 \overline{) 96} \end{array}$$

Limit numbers to NO remainders in the answer OR carried (each digit must be a multiple of the divisor).

$$\begin{array}{r} 18 \\ 4 \overline{) 732} \end{array}$$

Limit numbers to NO remainders in the final answer, but with remainders occurring within the calculation.

$$\begin{array}{r} 218 \\ 4 \overline{) 8732} \end{array}$$

Extend to 3-digit number first where the divisor can go into the first number and then progress to when the divisor cannot go into the first number.

$$\begin{array}{r} 037 \\ 5 \overline{) 1835} \end{array}$$

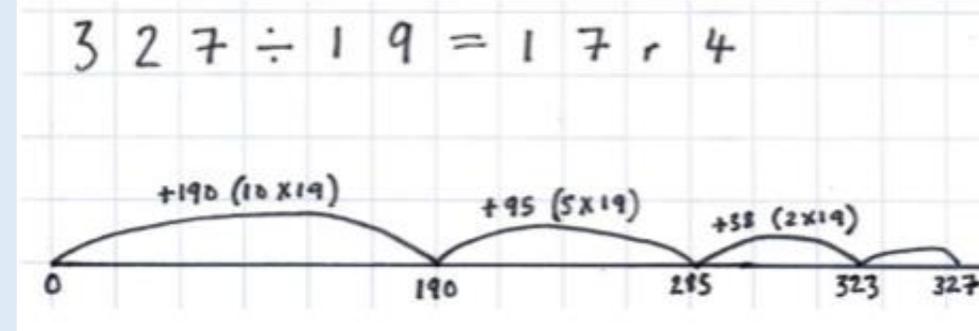
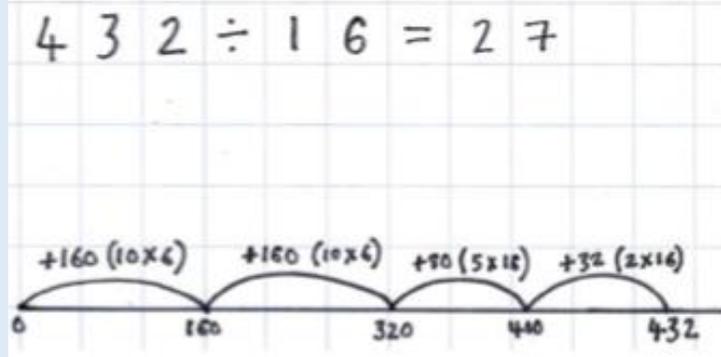
Remember to develop connections between fractions and division and rephrase these calculations as $\frac{1}{3}$ of 96; $\frac{1}{4}$ of 72, $\frac{1}{4}$ of 872 and $\frac{1}{5}$ of 185. Note: Year 3 fraction objective - *Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators*; Year 4 fraction objective: *solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number.*



Division: Year 5

Continue to use number lines to support the children's understanding of efficient grouping or the 'chunking' of multiples of the divisor.

Year 5 statutory requirement:
 ✓ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.



Further secure pupils' understanding of compact short division.

$$218 \div 8 = 27 \text{ r } 2$$

$$8 \overline{) 2158}$$

$96 \div 4 = 24$

$$\begin{array}{r} 24 \\ 4 \overline{) 96} \\ \underline{-40} (10 \times 4) \\ 56 \\ \underline{-40} (10 \times 4) \\ 16 \\ \underline{-16} (4 \times 4) \\ 00 \end{array}$$

The vertical chunking method is introduced but only with a single digit divisor. The number of groups should be recorded alongside on the right with the answer written on top of the bus stop. When this is secure, the same layout is used with remainders.

$43 \div 3 =$

$$\begin{array}{r} 14 \text{ r } 1 \\ 3 \overline{) 43} \\ \underline{-30} (10 \times 3) \\ 13 \\ \underline{-12} (4 \times 3) \\ 01 \end{array}$$

Extend to expressing results in different ways according to the context, including with remainders as fractions, as decimals or by rounding. For example:

- Whole number remainder = $27 \text{ r } 2$
- Fraction remainder = $27 \frac{2}{8} = 27 \frac{1}{4}$
- Decimal remainder = $27 \frac{1}{4} = 27 \frac{25}{100} = 27.25$



Division: Year 6



Year 6 statutory requirement:

✓ divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

Continue to use compact short division to divide numbers up to 4 digits by a 1-digit whole number.

Continue to develop the formal 'chunking' method to divide 3- or 4-digit numbers by a 2-digit whole number.

$$218 \div 8 =$$

$$27 \text{ r } 2$$

$$8 \overline{) 2158}$$

- Whole number remainder = **27 r 2**
- Fraction remainder = $27\frac{2}{8} = 27\frac{1}{4}$
- Decimal remainder = $27\frac{1}{4} = 27\frac{25}{100} = 27.25$

$$432 \div 16 = 27$$

$$\begin{array}{r} 27 \\ 16 \overline{) 432} \\ \underline{-160} \quad (10 \times 16) \\ 272 \\ \underline{-160} \quad (10 \times 16) \\ 112 \\ \underline{-80} \quad (5 \times 16) \\ 32 \\ \underline{-32} \quad (2 \times 16) \\ 00 \end{array}$$

$$327 \div 19$$

$$\begin{array}{r} 17 \text{ r } 4 \\ 19 \overline{) 327} \\ \underline{-190} \quad (10 \times 19) \\ 137 \\ \underline{-95} \quad (5 \times 19) \\ 38 \\ \underline{-38} \quad (2 \times 19) \\ 04 \end{array}$$

$$\begin{array}{r} 23 \\ 19 \overline{) 437} \\ \underline{-38} \quad \downarrow \\ 057 \\ \underline{-57} \\ 00 \end{array}$$

$$\begin{array}{r} 024 \text{ r } 12 \\ 24 \overline{) 588} \\ \underline{-48} \\ 108 \\ \underline{-96} \\ 12 \end{array}$$

If appropriate, the formal long division method can be introduced to divide numbers up to 4 digits by a 2-digit whole number.